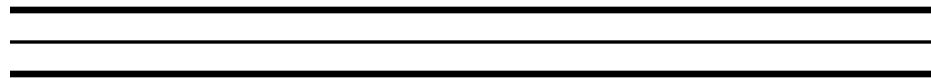


# NANOINDENTATION AND THE DYNAMIC CHARACTERIZATION OF VISCOELASTIC SOLIDS

*E. G. Herbert<sup>1</sup>, W. C. Oliver<sup>1</sup>, and G. M. Pharr<sup>2</sup>*

*<sup>1</sup> University of Tennessee, Dept. of Materials Science and Engineering;  
& MTS Nano Instruments Innovation Center*

*<sup>2</sup> University of Tennessee, Dept. of Materials Science and Engineering;  
& Oak Ridge National Laboratory, Metals and Ceramics Division*

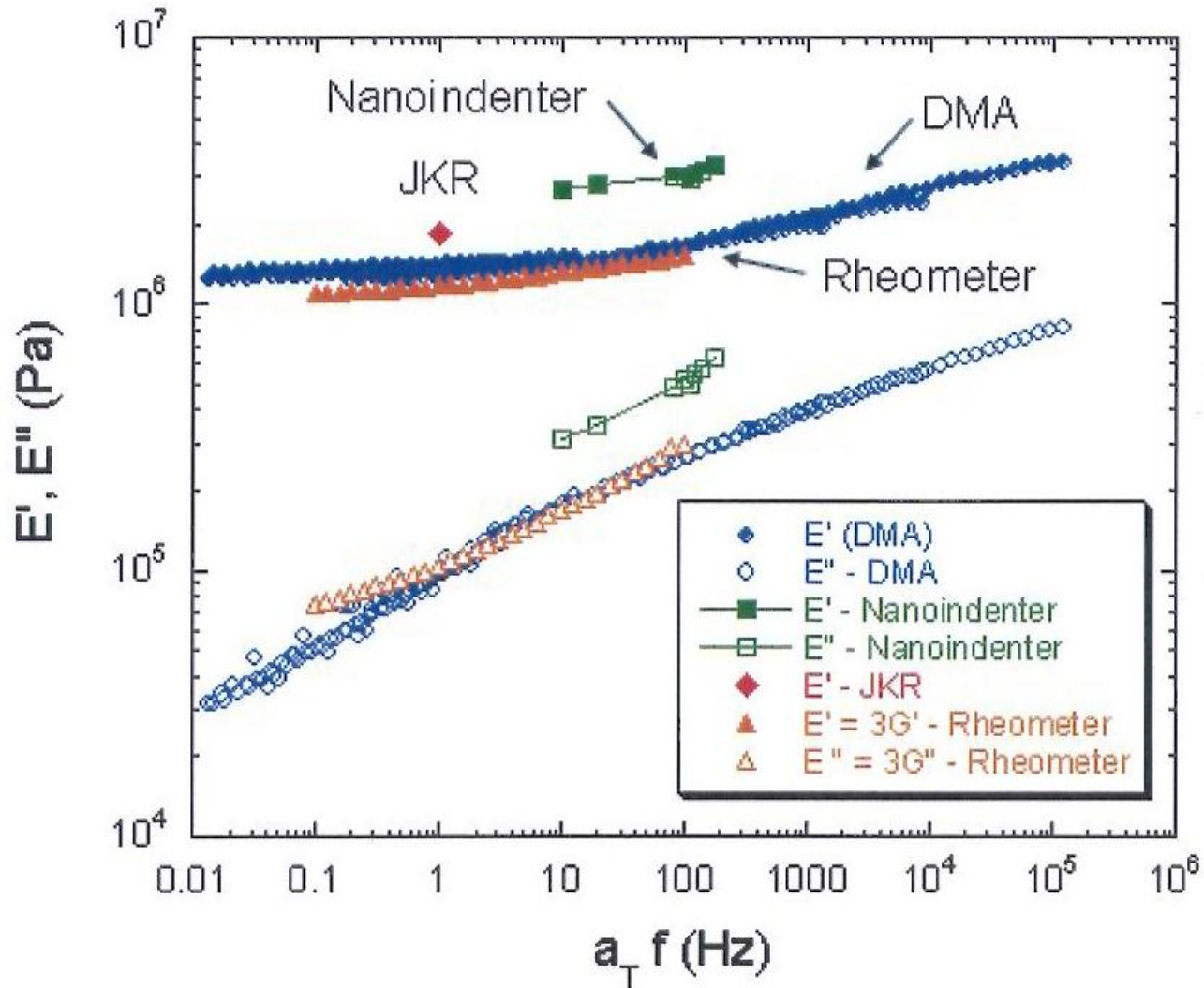


# MOTIVATION

- ❖ ***Extend the applicability of nanoindentation to time-dependent behavior***
  - Viscoelastic functions and ***dynamic behavior***
  - ***Complex modulus***
- What we're after:
  - Constitutive behavior of small volumes of viscoelastic solids subjected to sinusoidal loading

$$\sigma = \varepsilon_0 E' \sin \omega t + \varepsilon_0 E'' \cos \omega t$$

# NANOINDENTATION & DMA COMPARISON



C. C. White et al., Mater. Res. Soc. Symp. Proc. **841** (2005)



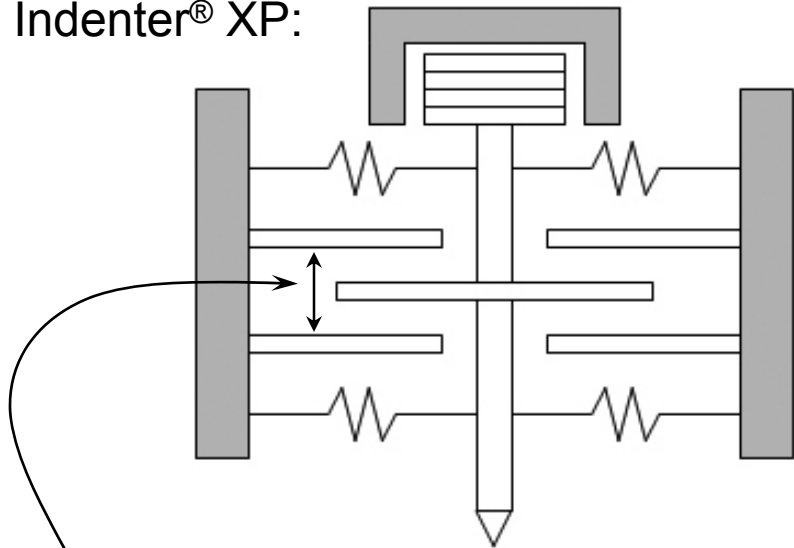
# MODELING THE INSTRUMENTATION

- Measure time-dependent material properties
- Then we need to understand the time-dependent properties of the measurement tool

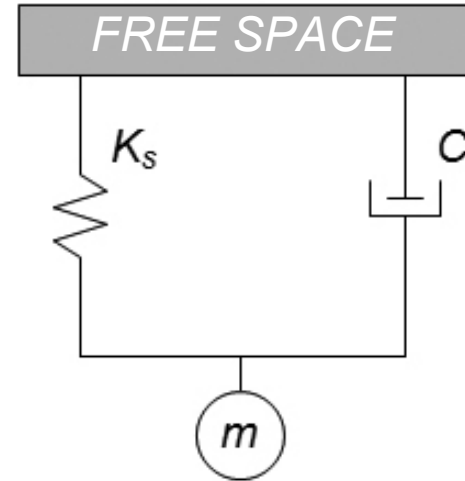


# MODELING THE INSTRUMENTATION

Nano Indenter<sup>®</sup> XP:



Raw displacement,  $\pm 1$  mm



$$K_s = \frac{F_o}{h_o} \cos \phi + m\omega^2$$

$$C = \frac{F_o}{h_o} \frac{\sin \phi}{\omega}$$

$$F_o e^{i\omega t} = m\ddot{h} + C\dot{h} + Kh$$

$$h(t) = h_o e^{i(\omega t - \phi)}$$

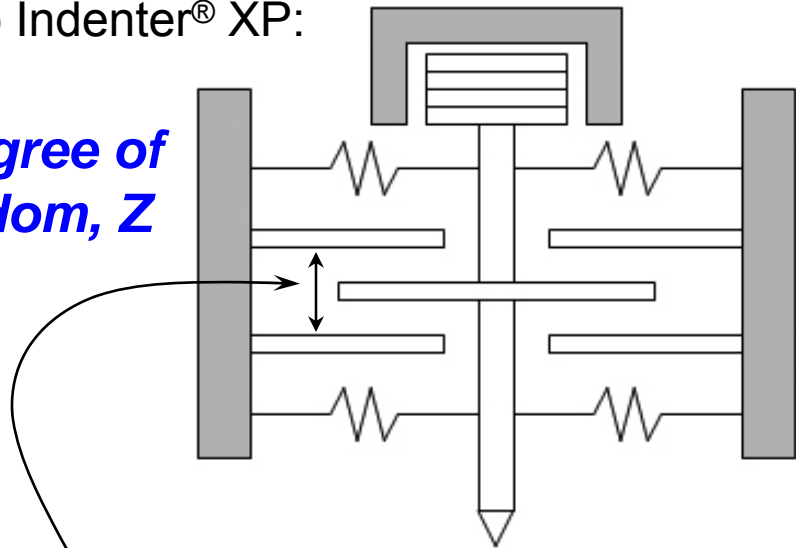
$$\frac{h_o}{F_o} = \left[ \left( (K - m\omega^2)^2 + \omega^2 C^2 \right)^{1/2} \right]^{-1}$$

$$\tan \phi = \frac{C\omega}{K - m\omega^2}$$

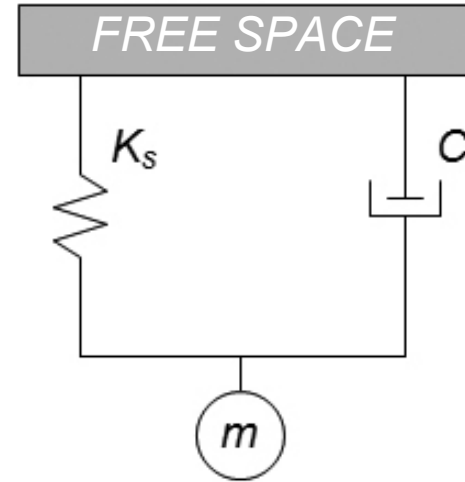
# MODELING THE INSTRUMENTATION

Nano Indenter<sup>®</sup> XP:

1 degree of freedom, Z



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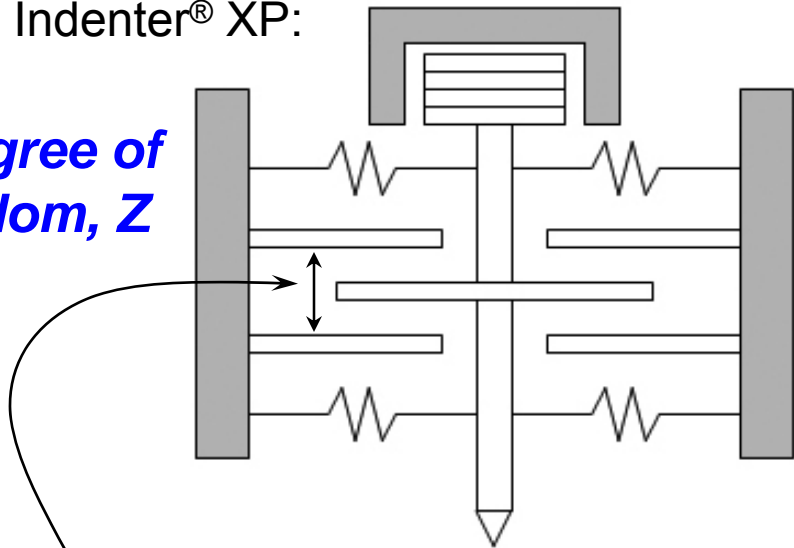
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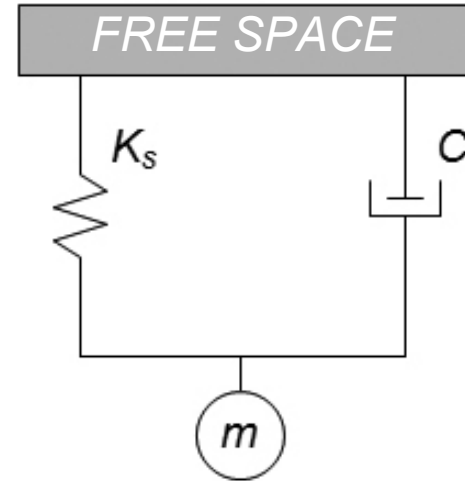
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**FUNCTION OF POSITION**

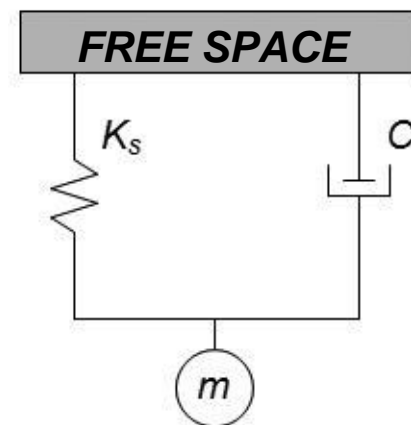
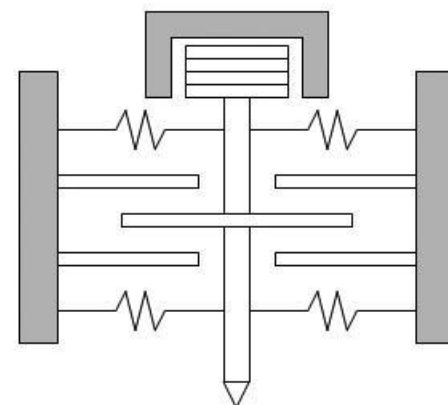
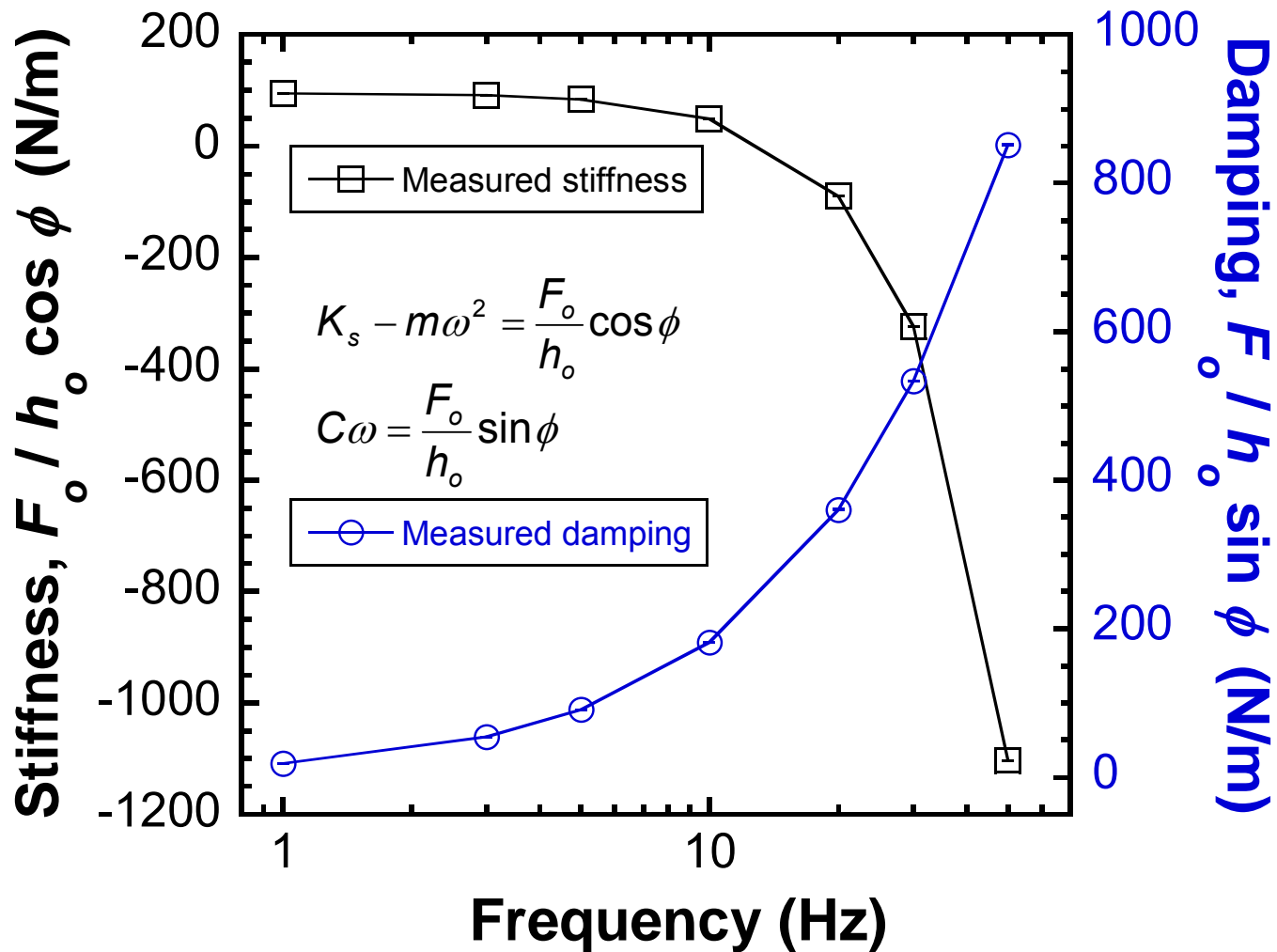
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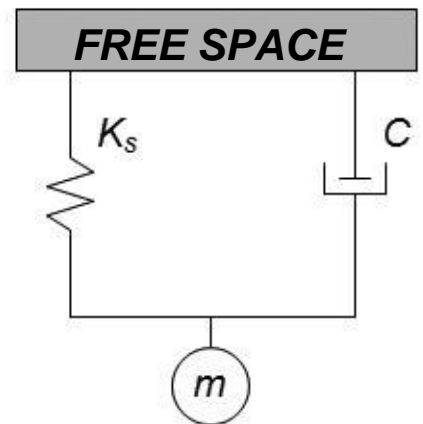
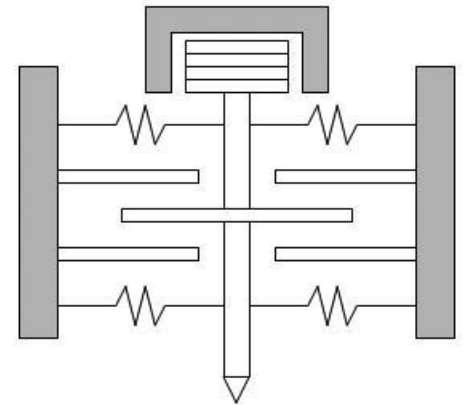
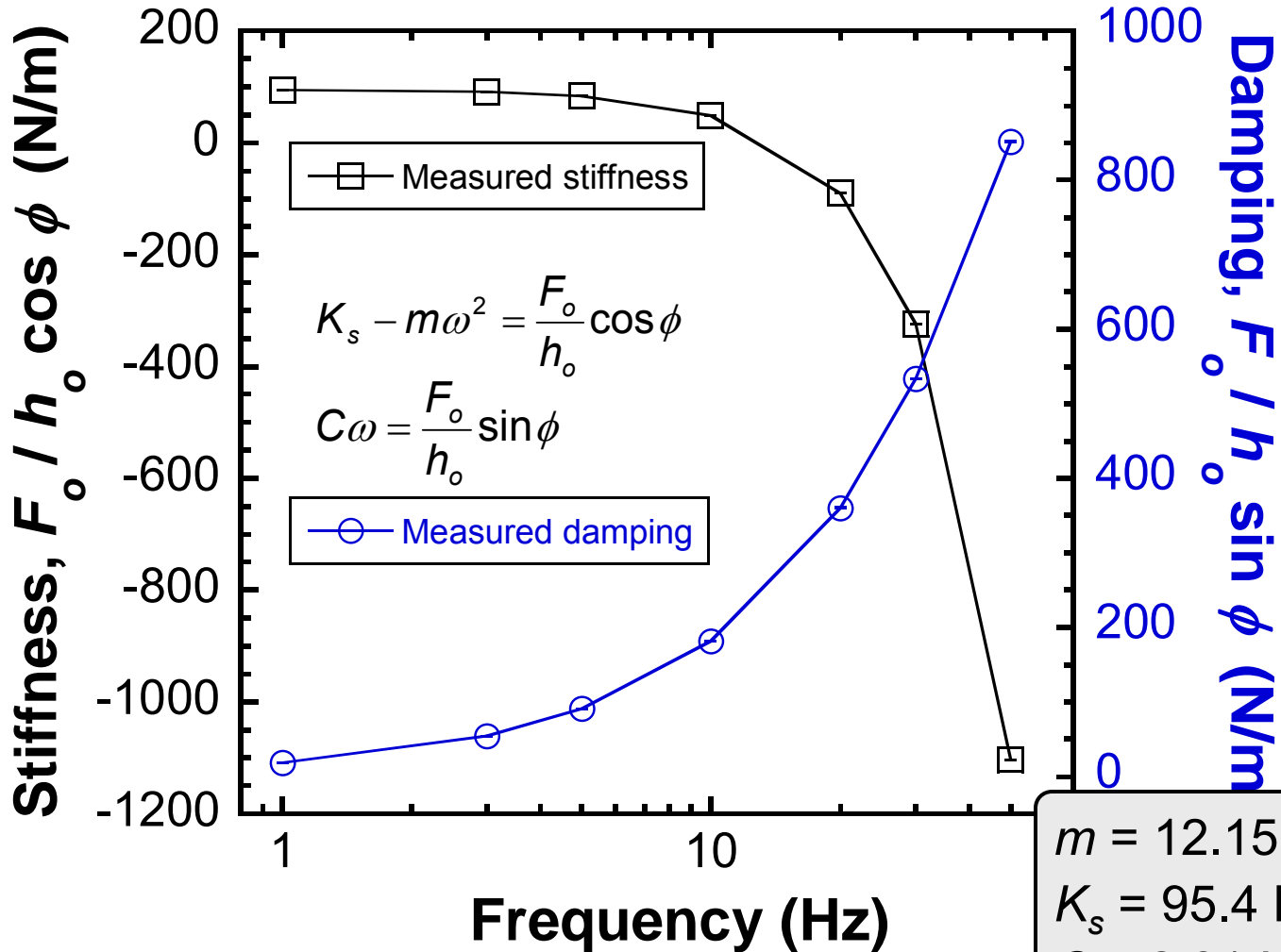
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# Measured stiffness and damping in free space, position = 18.8 $\mu\text{m}$



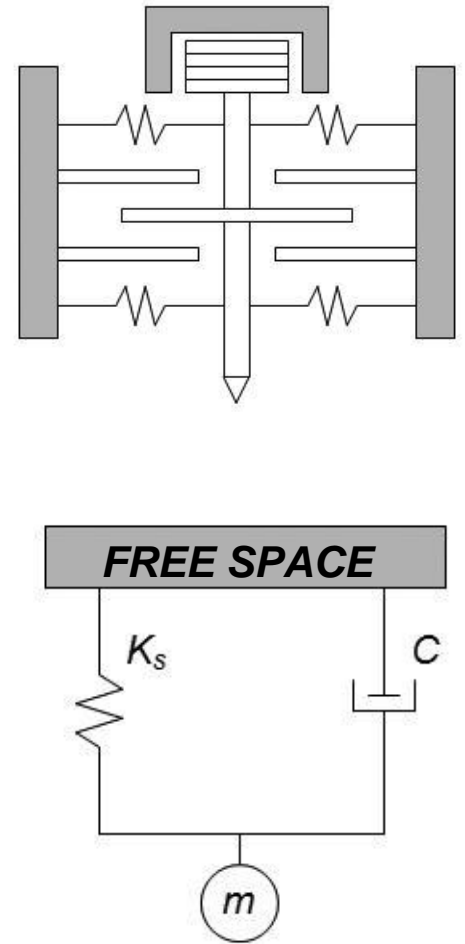
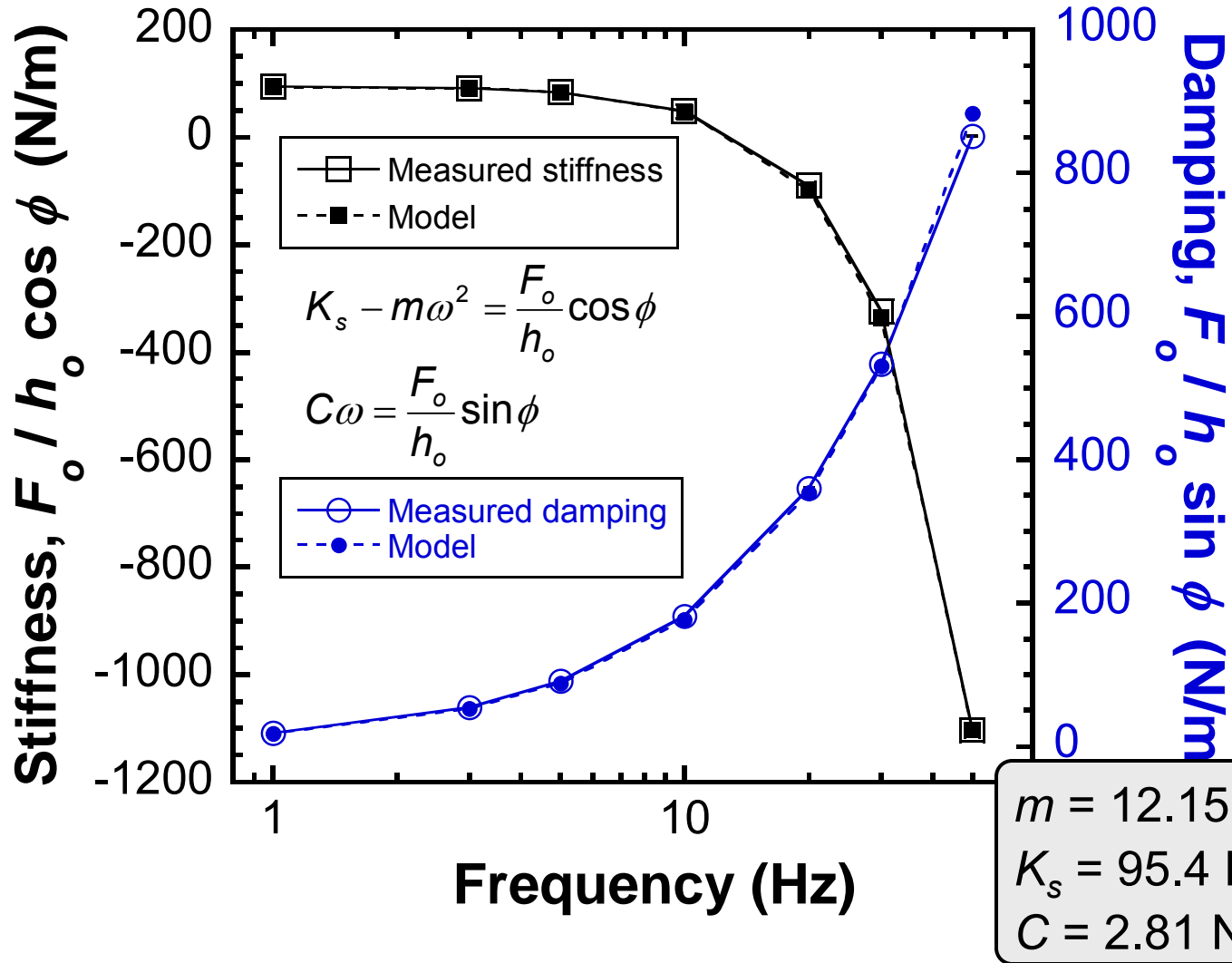
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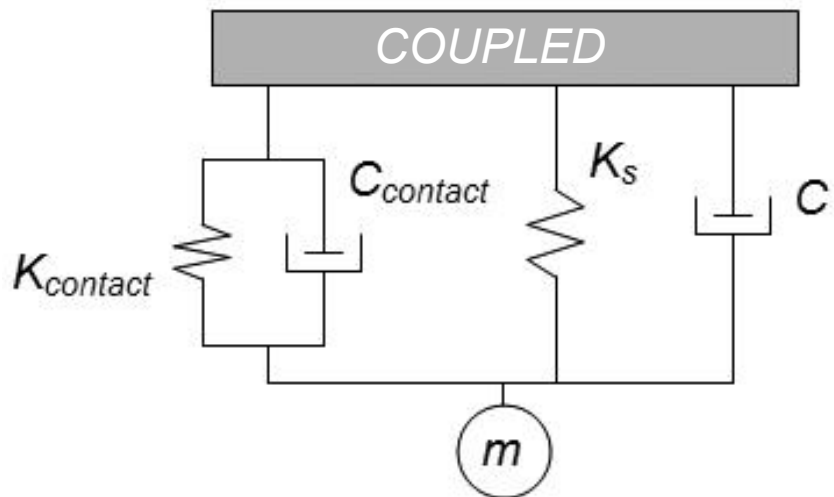
$m = 12.15 \text{ g}$   
 $K_s = 95.4 \text{ N/m}$   
 $C = 2.81 \text{ Ns/m}$



# Measured stiffness and damping in free space, position = 18.8 μm



# ADD THE CONTACT



$$S = \frac{F_o}{h_o} \cos \phi + m\omega^2$$

$$C = \frac{F_o}{h_o} \frac{\sin \phi}{\omega}$$

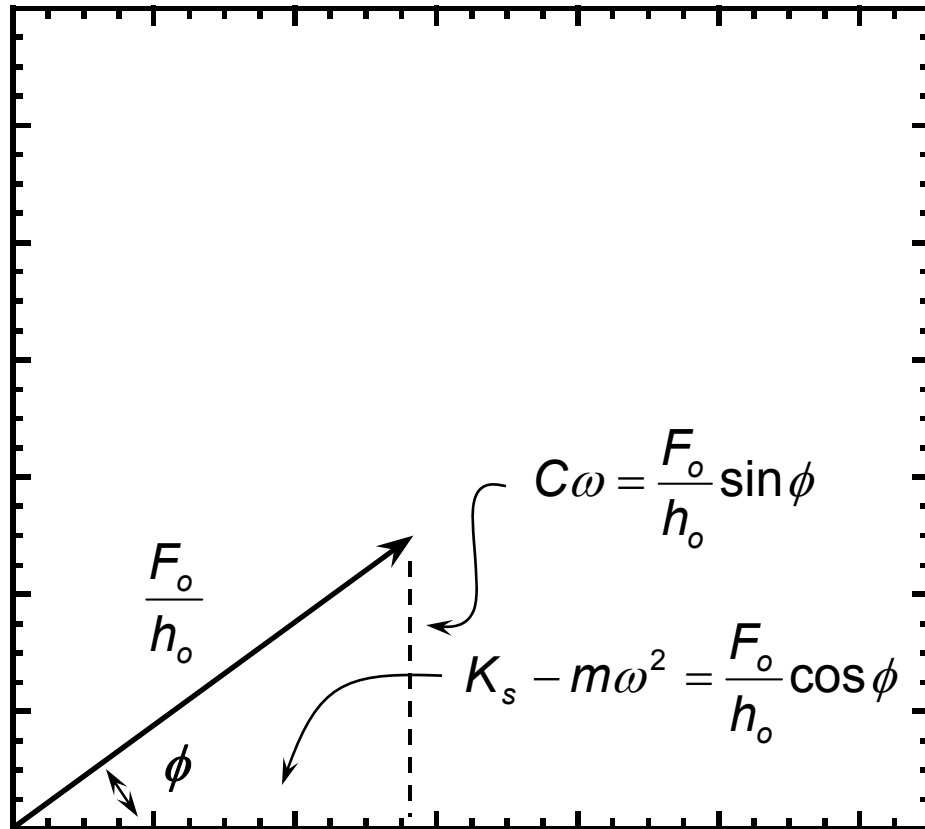
**COUPLED RESPONSE = SAMPLE + INSTRUMENT**

$$K_{contact} = \left[ \frac{F_o}{h_o} \cos \phi + m\omega^2 \right]_{\text{coupled}} - \left[ \frac{F_o}{h_o} \cos \phi + m\omega^2 \right]_{\text{inst. (free space)}}$$

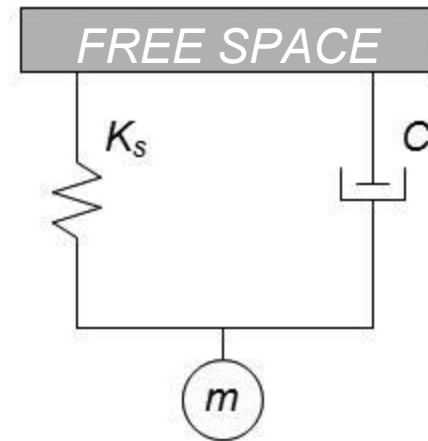
# PHASOR DIAGRAM: PHYSICAL INSIGHT

## Damped, forced oscillator

Imaginary axis (damping,  $C\omega$ , N/m)



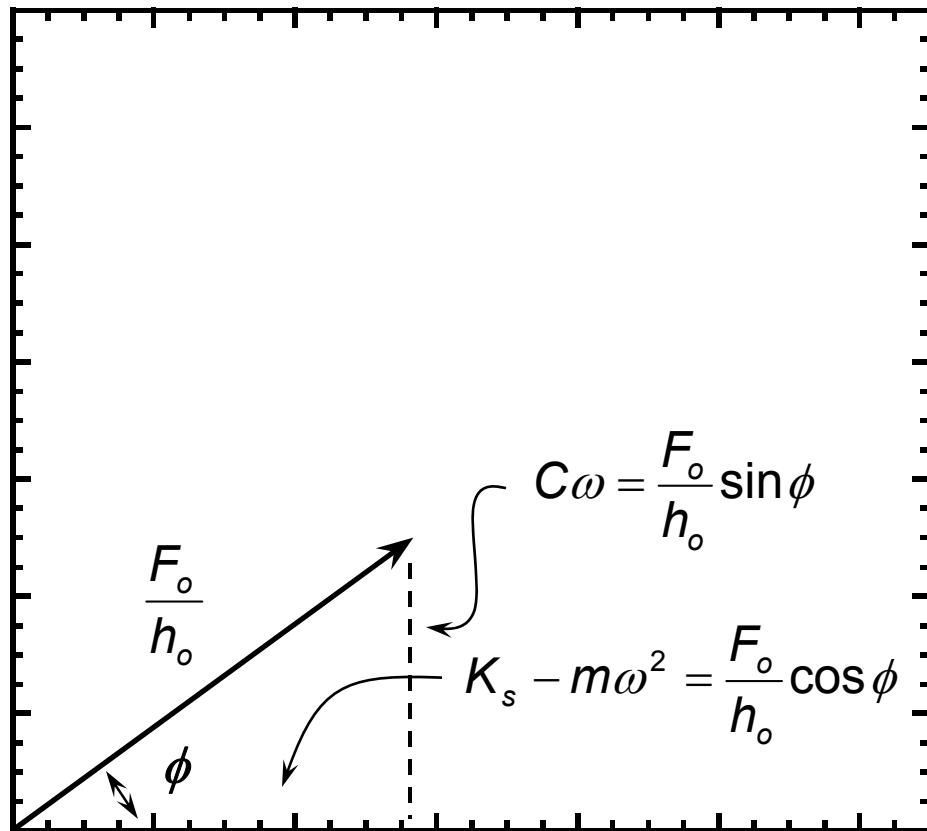
Real axis (stiffness,  $K_s - m\omega^2$ , N/m)



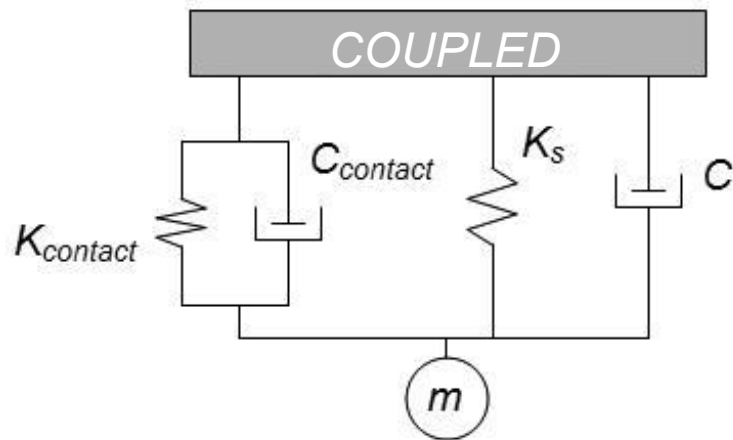
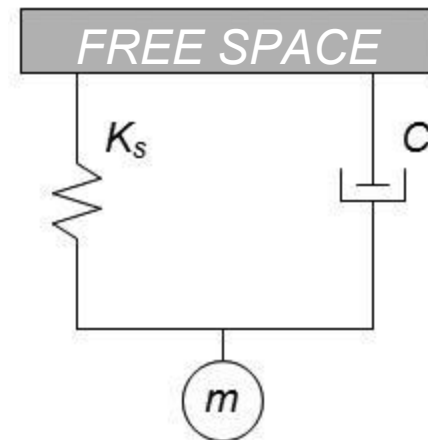
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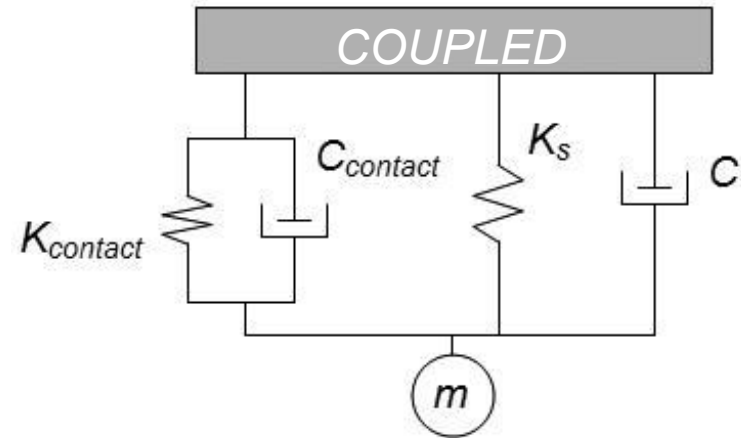
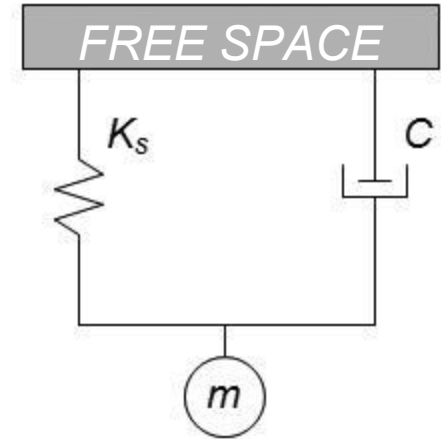
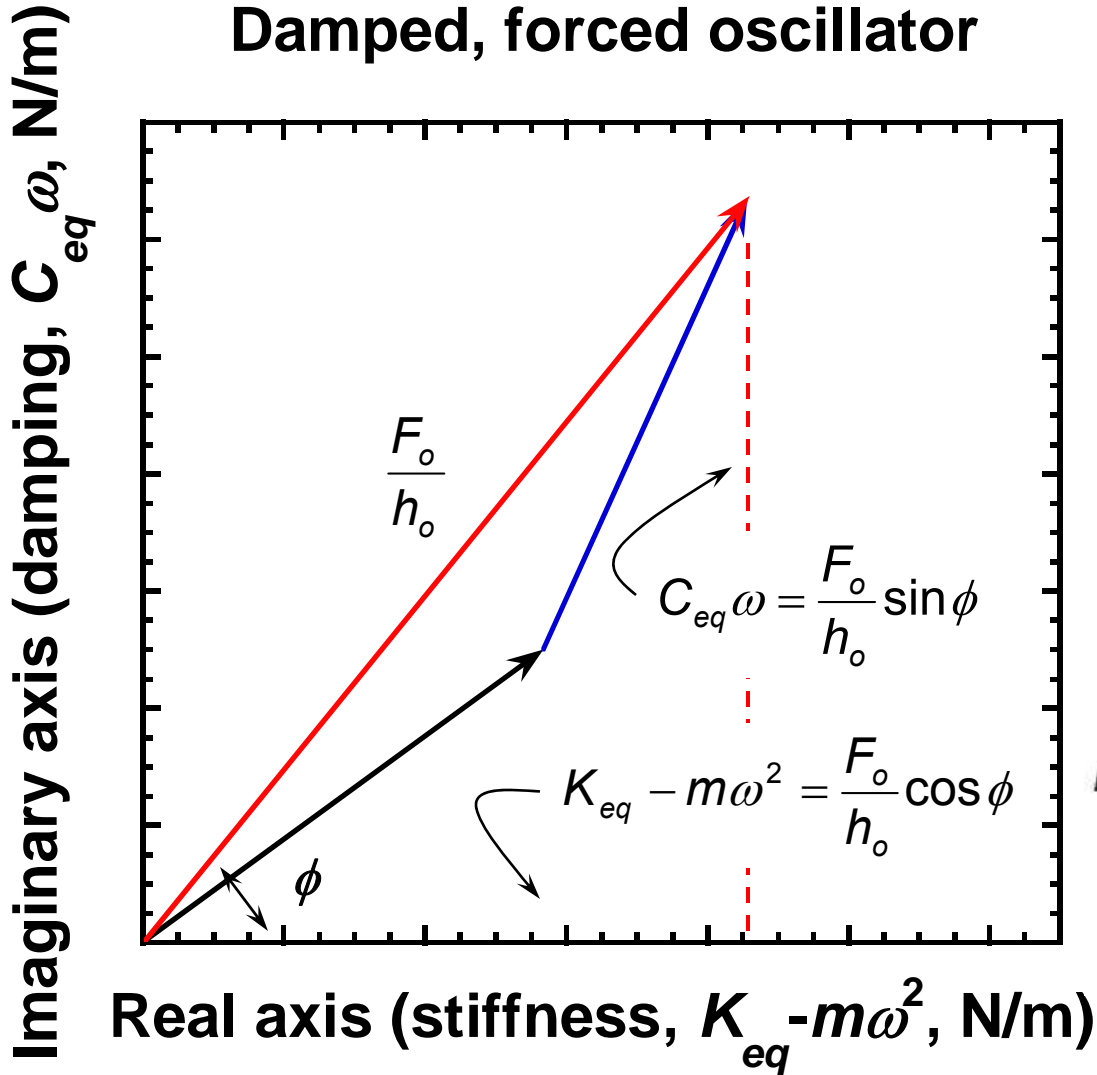


Real axis (stiffness,  $K_s - m\omega^2$ , N/m)



# PHASOR DIAGRAM: PHYSICAL INSIGHT

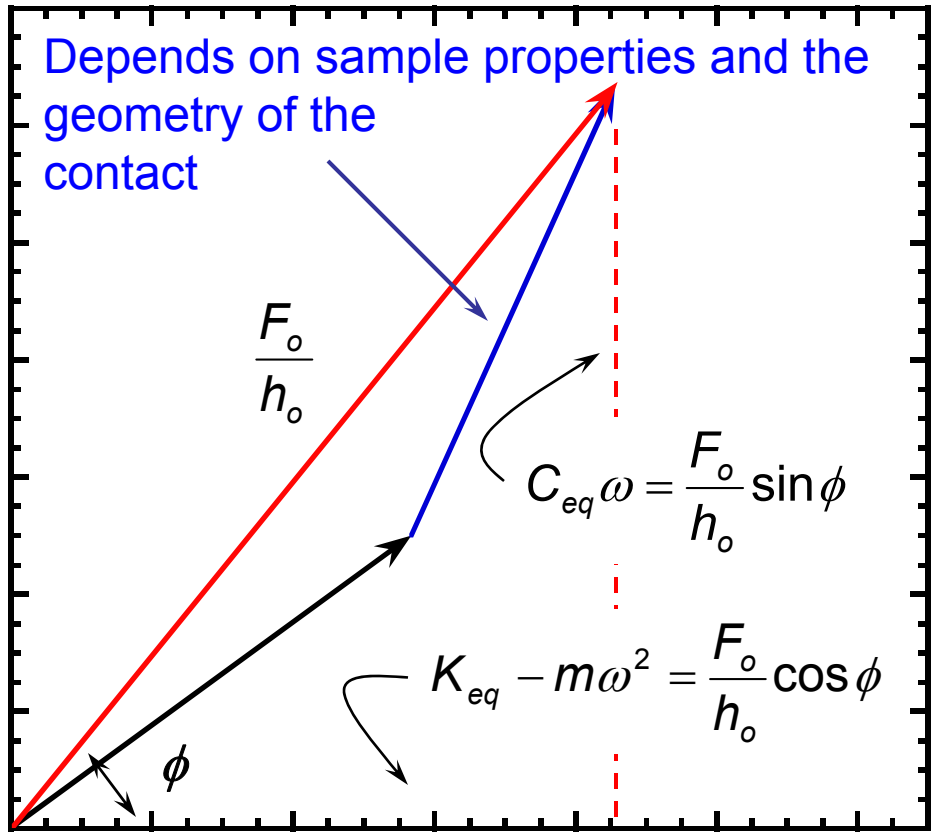
## Damped, forced oscillator



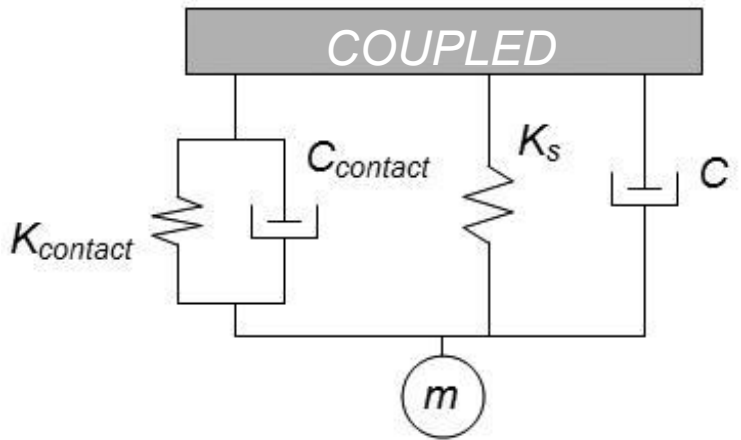
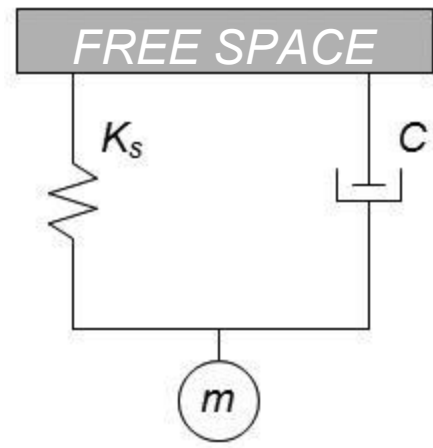
# PHASOR DIAGRAM: PHYSICAL INSIGHT

## Damped, forced oscillator

Imaginary axis (damping,  $C_{eq} \omega$ , N/m)



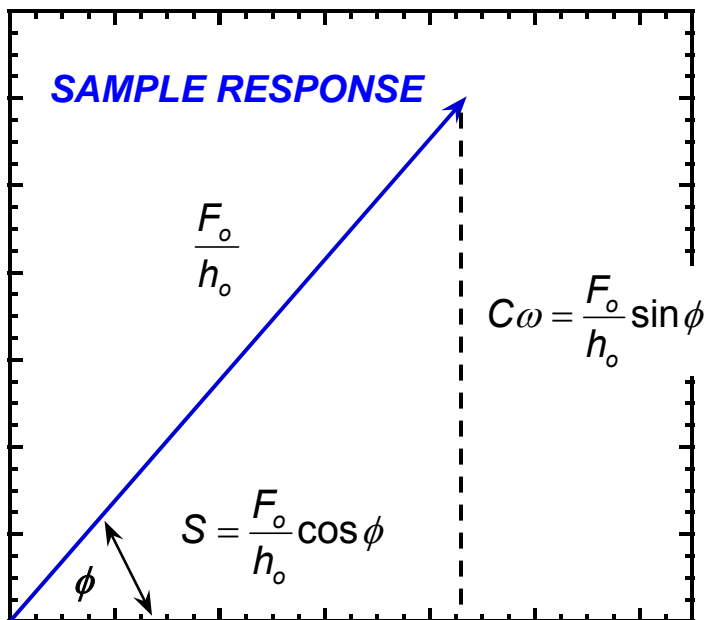
Real axis (stiffness,  $K_{eq} - m\omega^2$ , N/m)



# FROM S AND $C\omega \rightarrow E'$ AND $E''$

Phasor diagram of experimental measurements

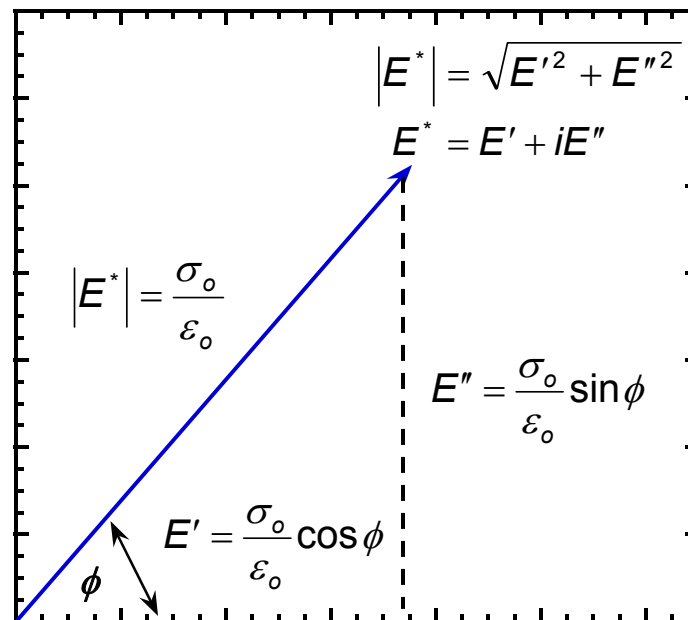
Imaginary axis (damping,  $C\omega$ , N/m)



Real axis (stiffness,  $S$ , N/m)

Phasor diagram of a linear viscoelastic solid

Imaginary axis (viscous stress, Pa)

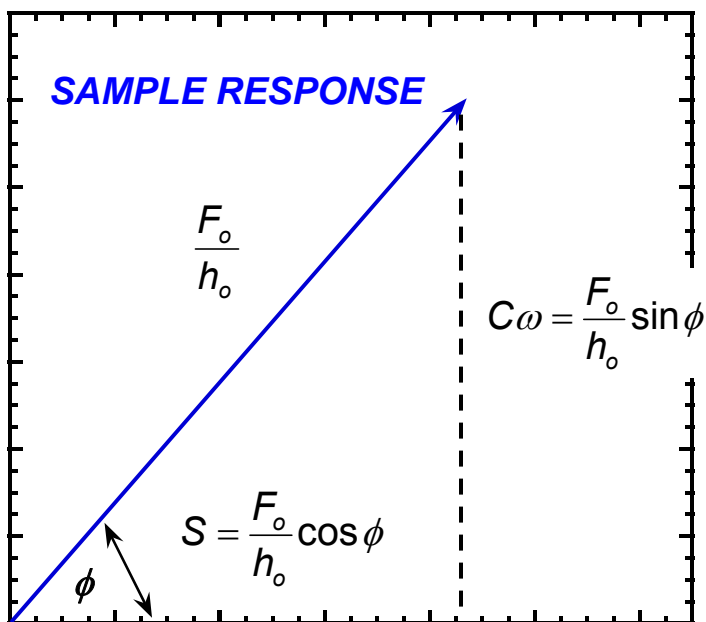


Real axis (elastic stress, Pa)

# FROM S AND $C\omega \rightarrow E'$ AND $E''$

Imaginary axis (damping,  $C\omega$ , N/m)

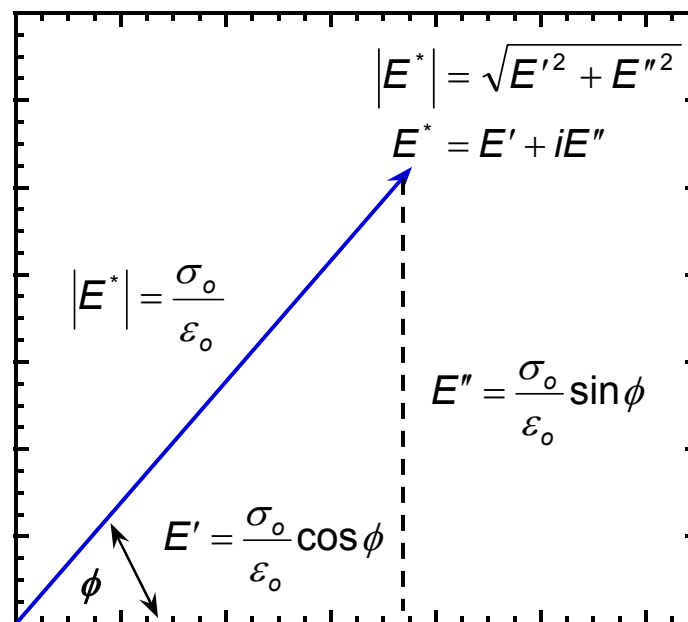
Phasor diagram of experimental measurements



Real axis (stiffness,  $S$ , N/m)

Imaginary axis (viscous stress, Pa)

Phasor diagram of a linear viscoelastic solid



Real axis (elastic stress, Pa)

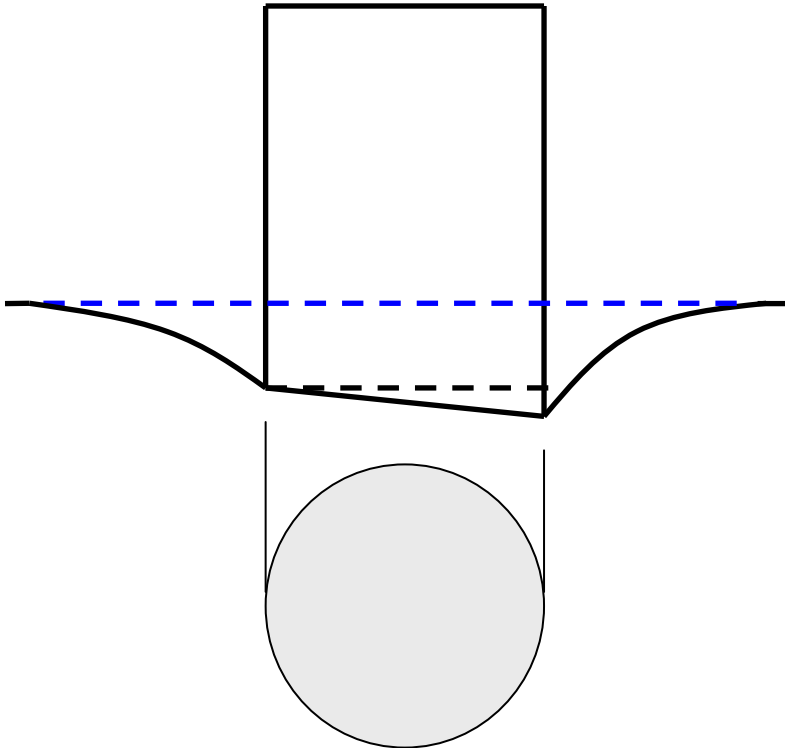
**The fundamental equation of nanoindentation:**

$$E' = S \frac{\sqrt{\pi}}{2} \frac{1}{\beta} \frac{1}{\sqrt{A}} \quad E'' = C\omega \frac{\sqrt{\pi}}{2} \frac{1}{\beta} \frac{1}{\sqrt{A}}$$



# GEOMETRY OF THE CONTACT

*Circular flat punch:*



## ***Advantages:***

- Known contact area
- Area not affected by creep or thermal drift

## ***Disadvantages:***

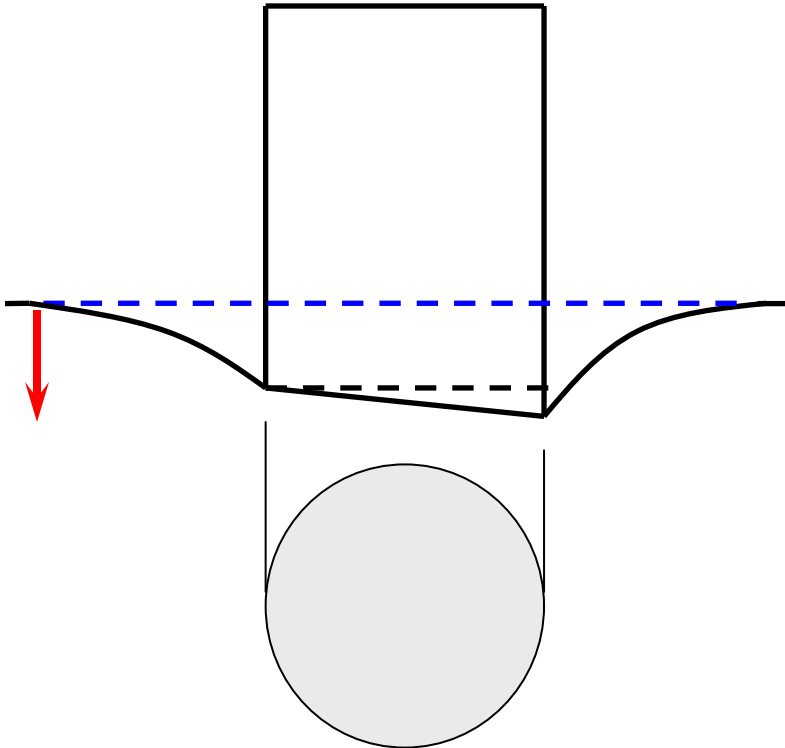
- Full contact
- Stress concentration

## ***Any tip geometry, consider:***

- Steady-state harmonic motion
- Linear viscoelasticity
  - Compression distance
  - Oscillation amplitude

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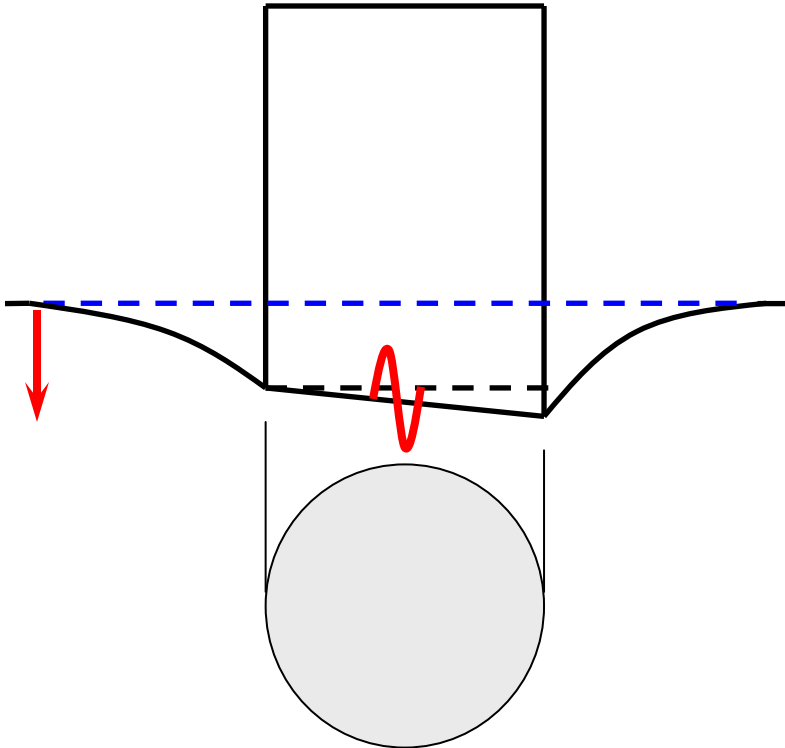
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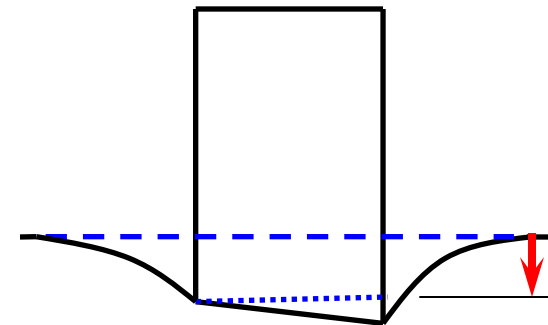
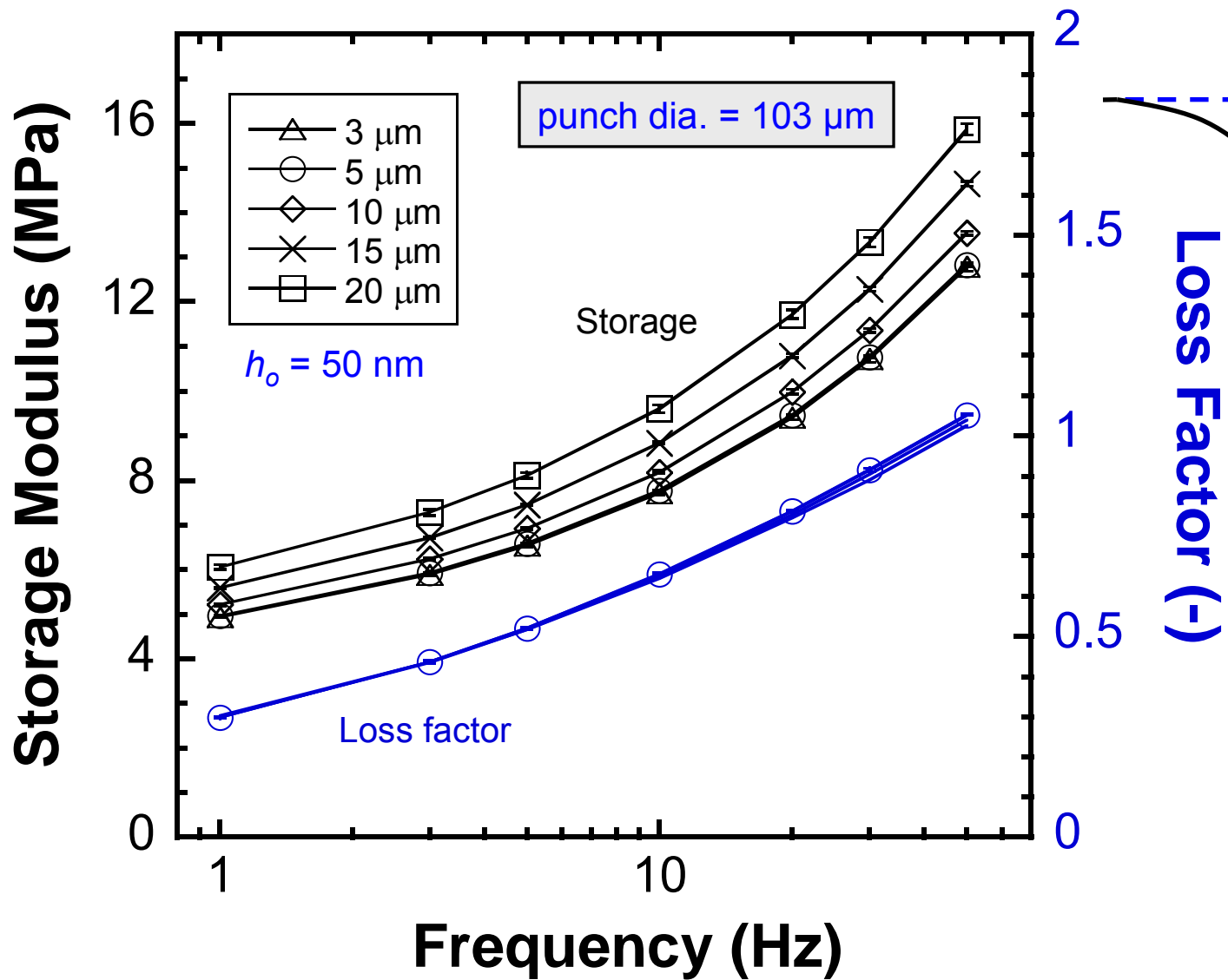
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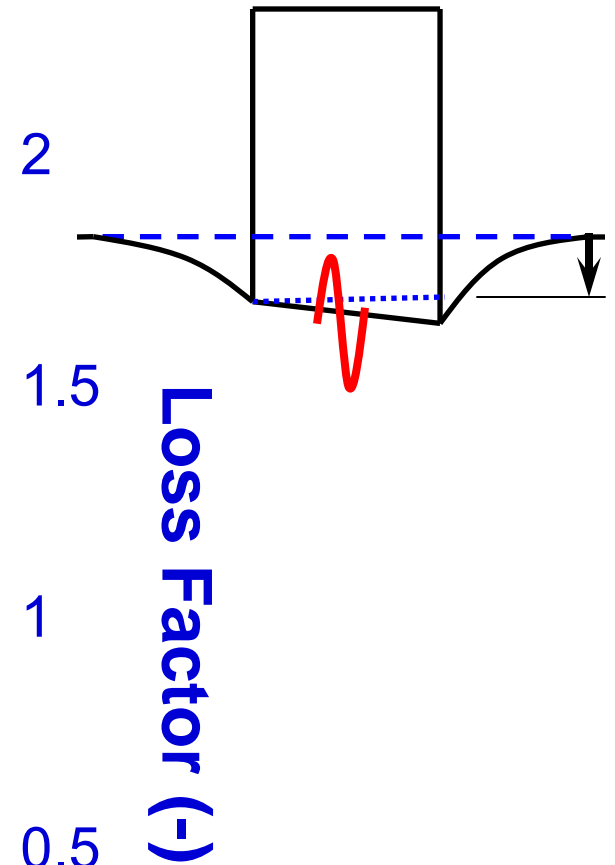
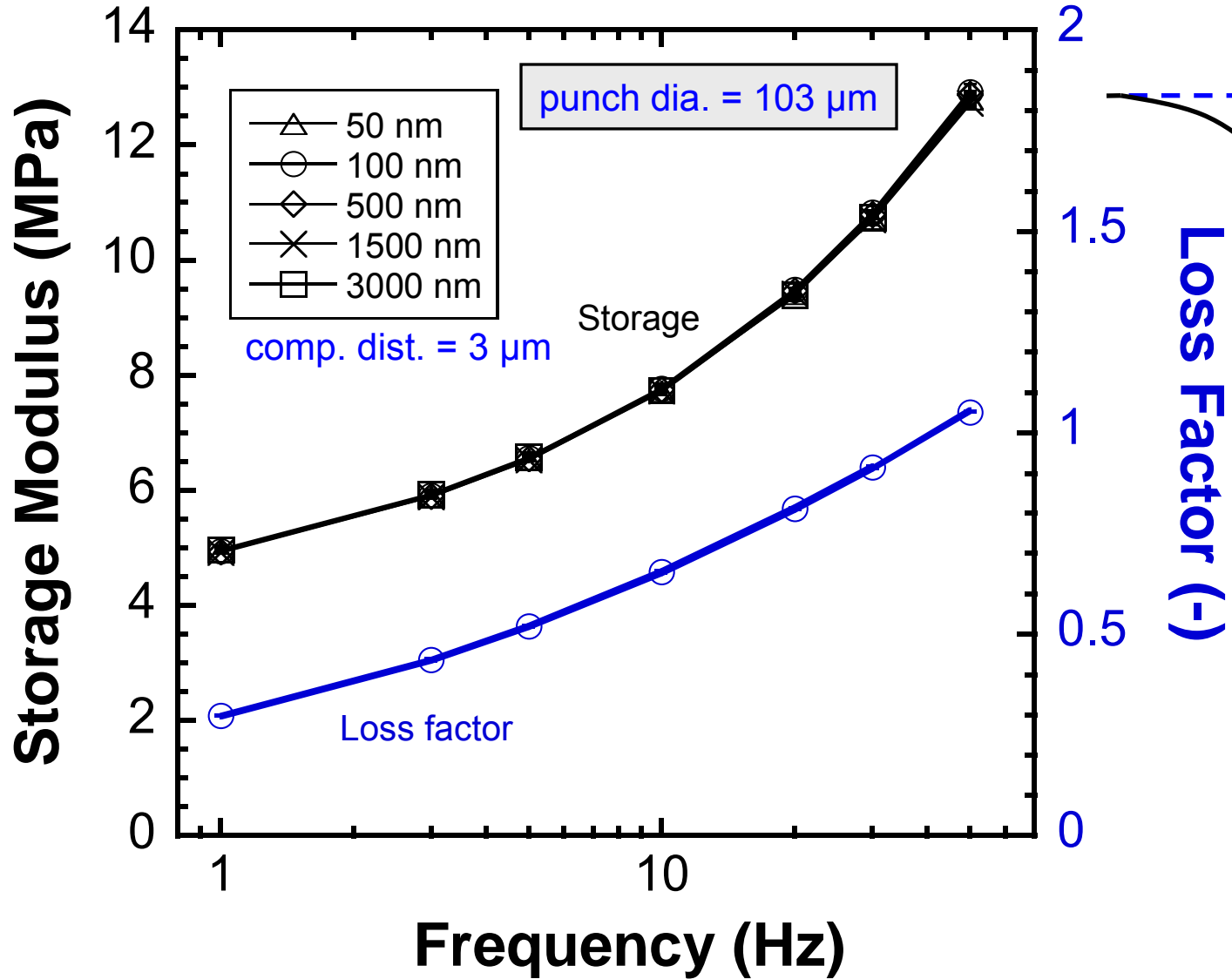
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# Compression Distance

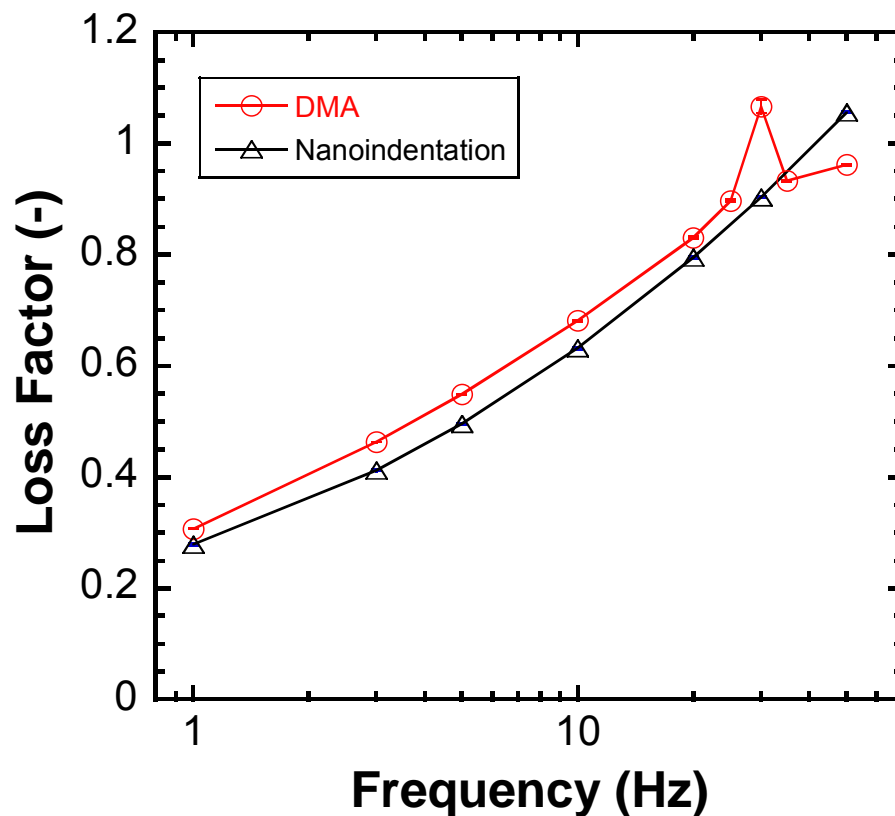
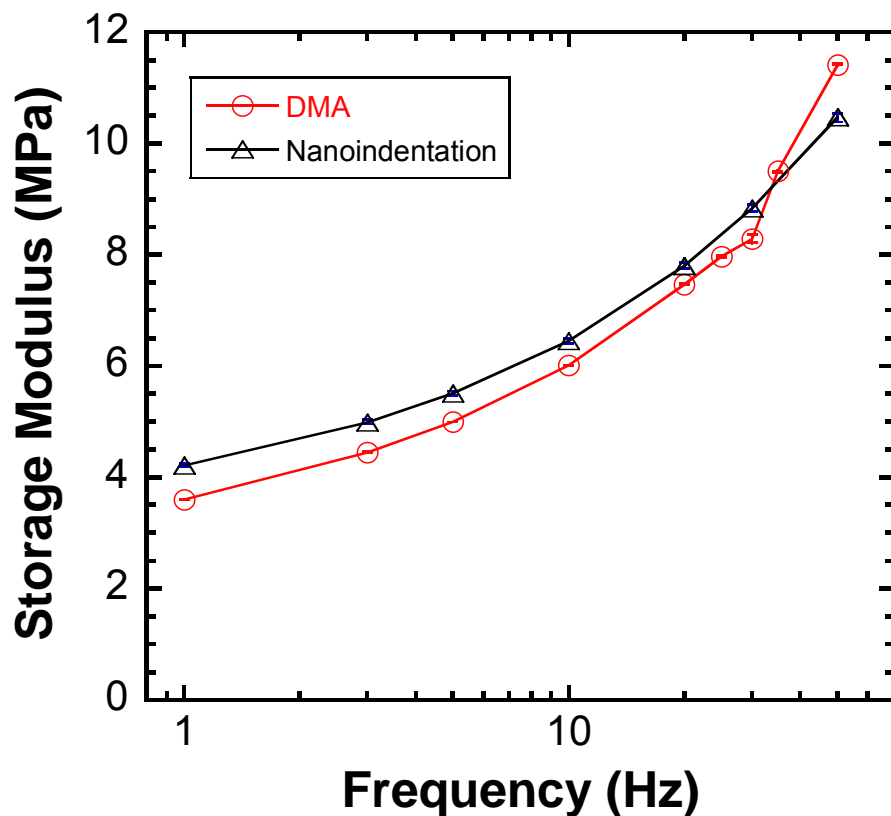


# Oscillation Amplitude



# DMA VS. NANOINDENTATION

Highly plasticized polyvinylchloride,  
the complex modulus at 22 °C

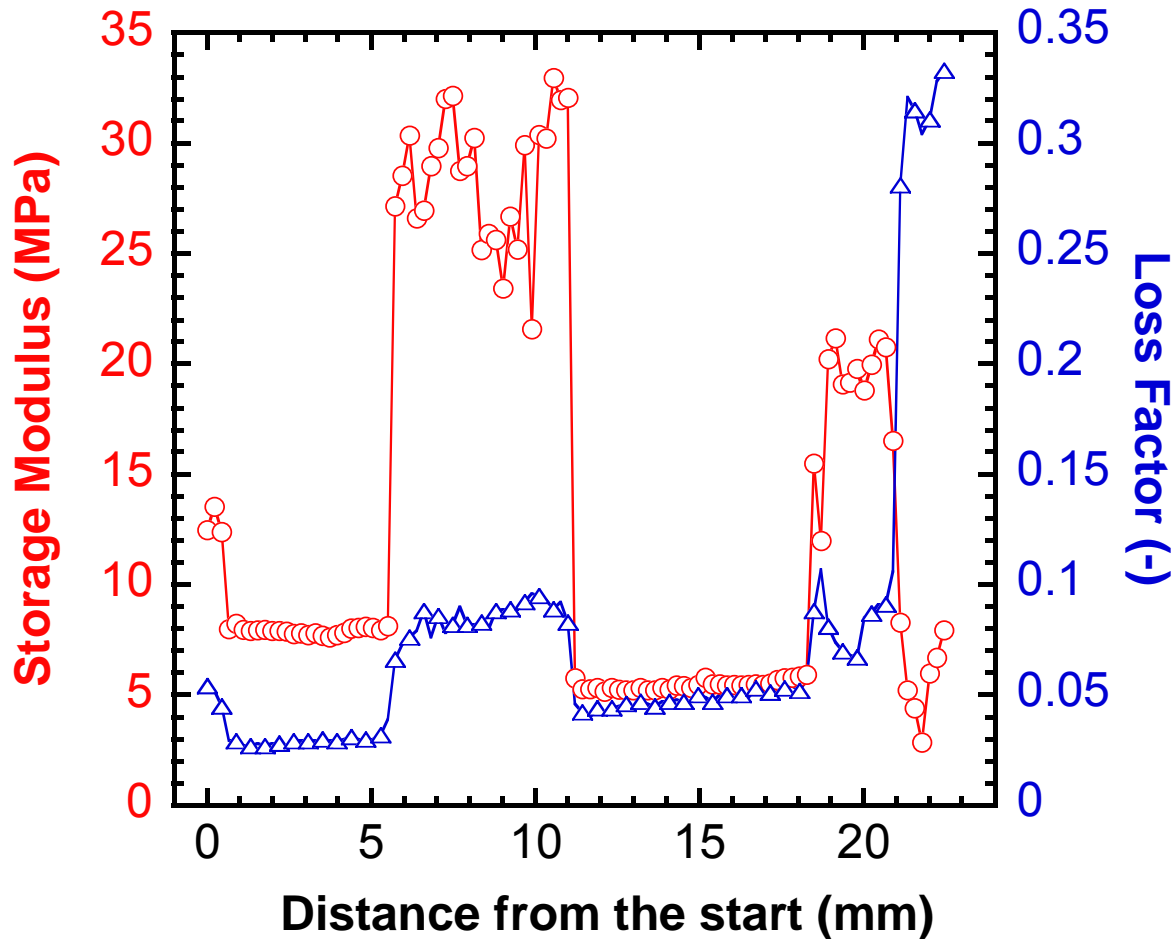


# APPLICATION: GOODYEAR TIRE AND RUBBER CO.

Courtesy Tom Fleischman and Remi Granier



Cross Profile



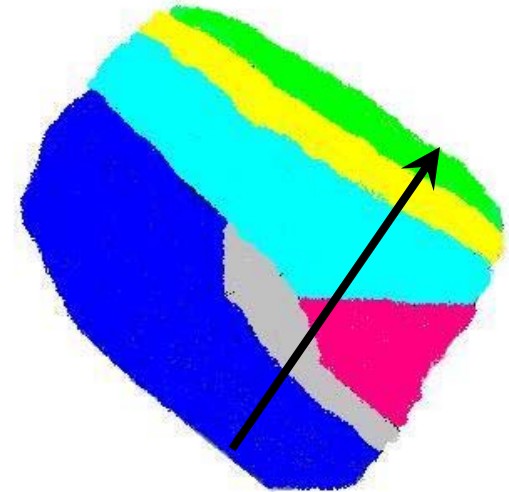
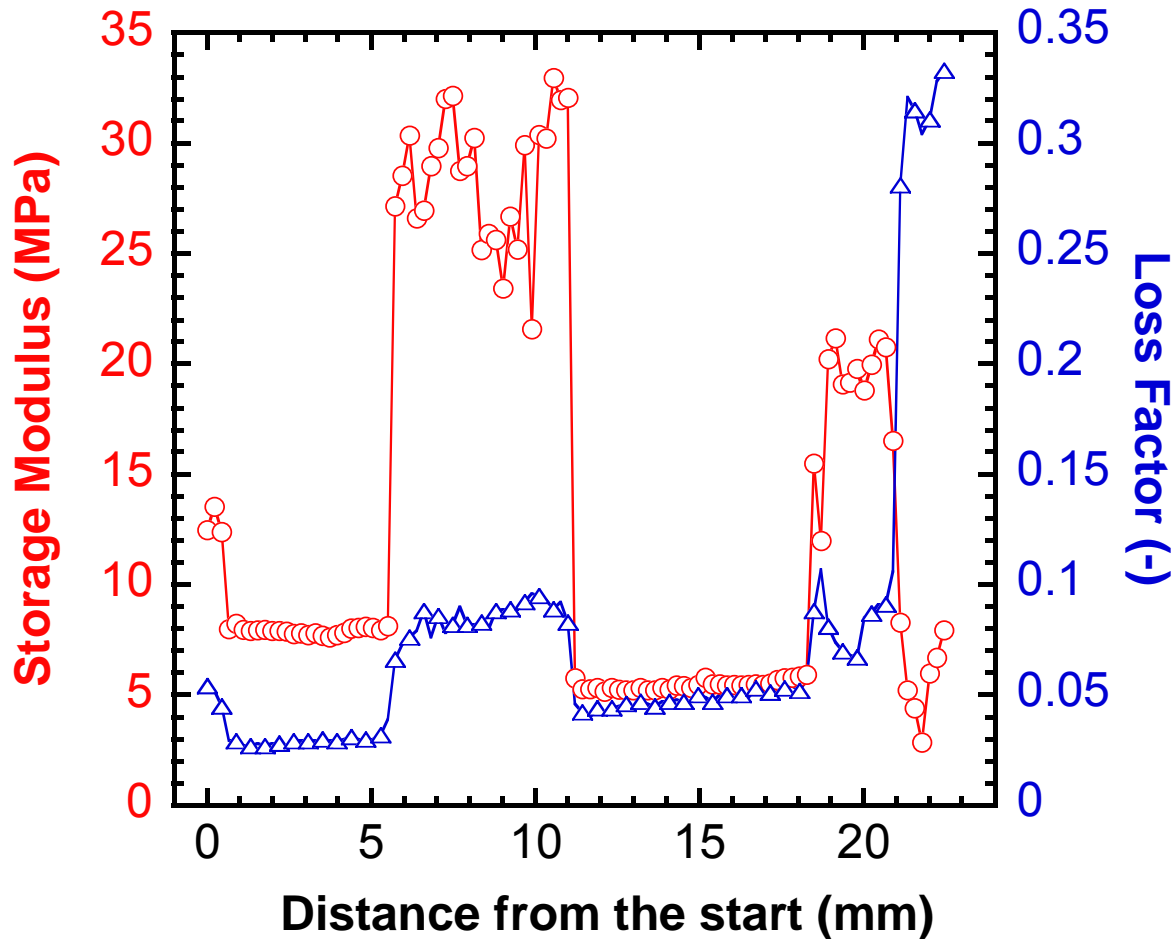
- Lab cure vs. actual
- 45  $\mu\text{m}$  dia. flat punch
- 10 Hz
- 103 measurements
- 90 seconds per test
- No failed tests



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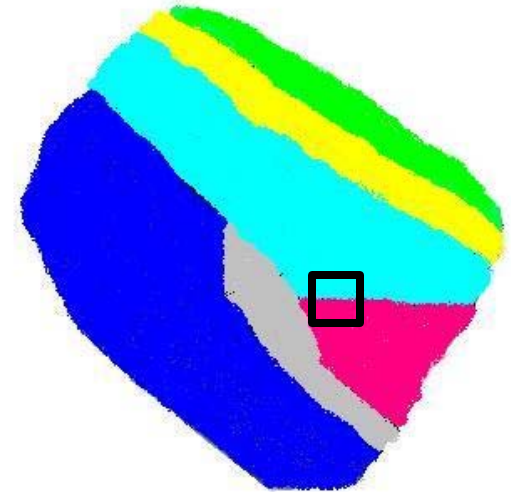
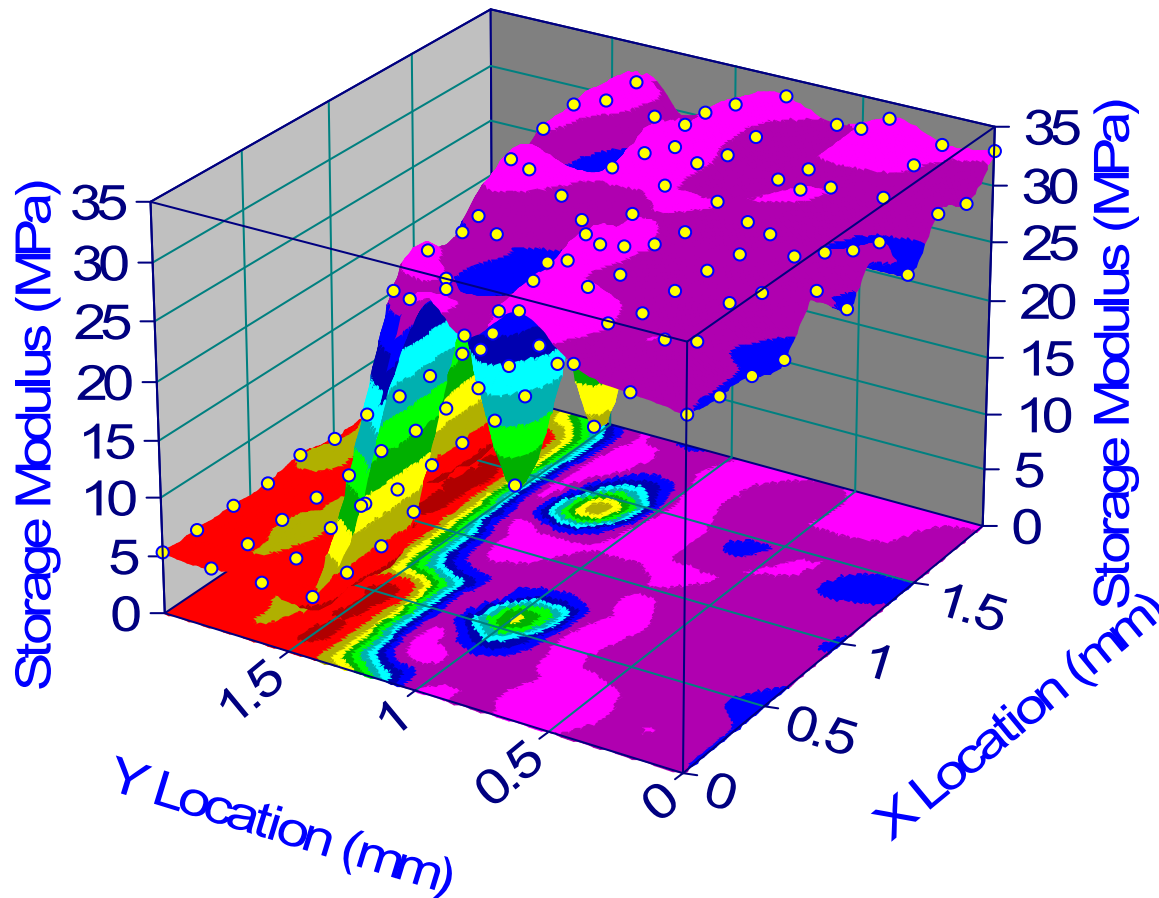


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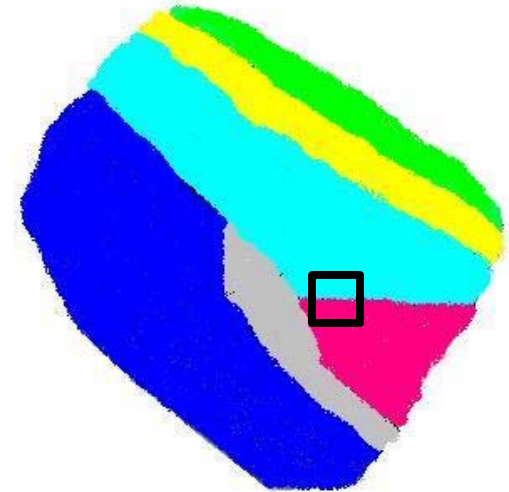
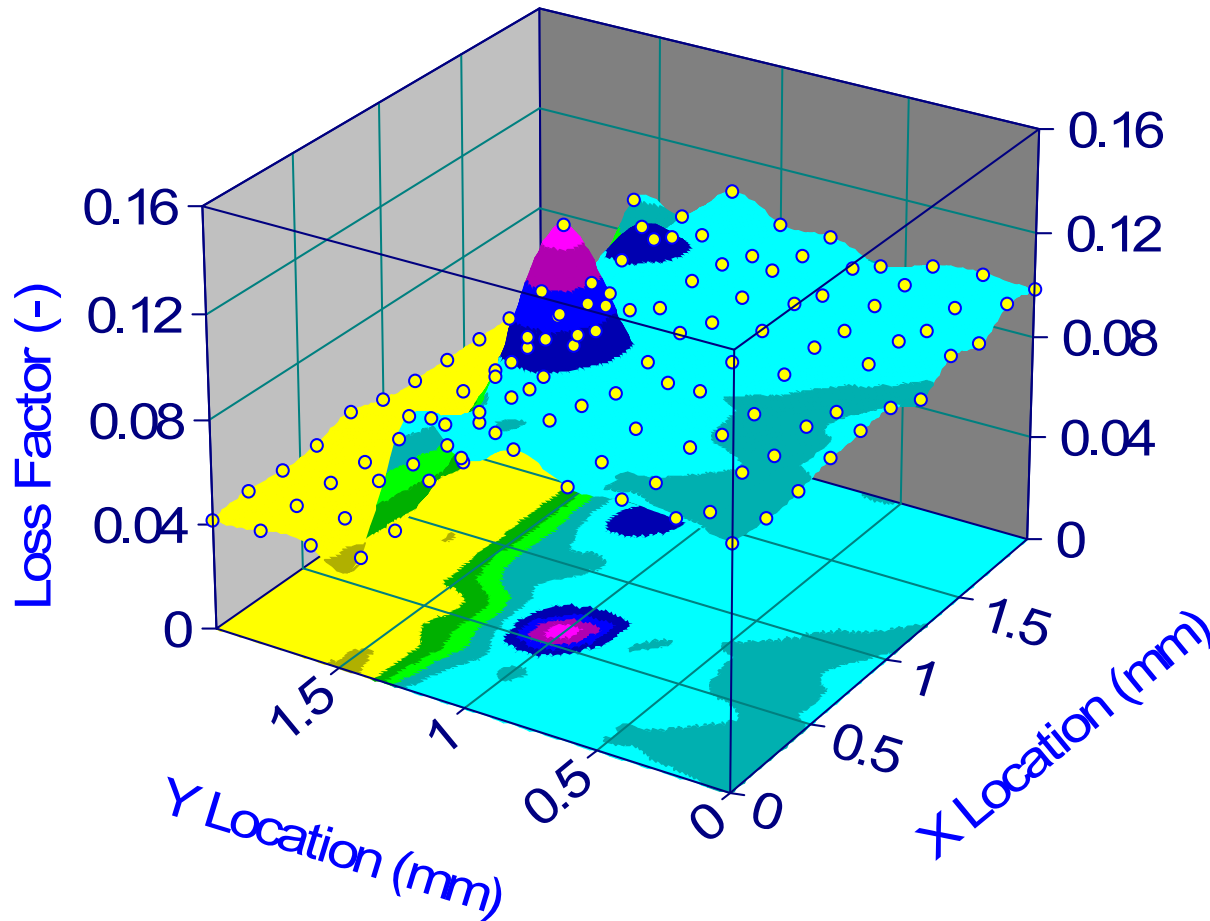


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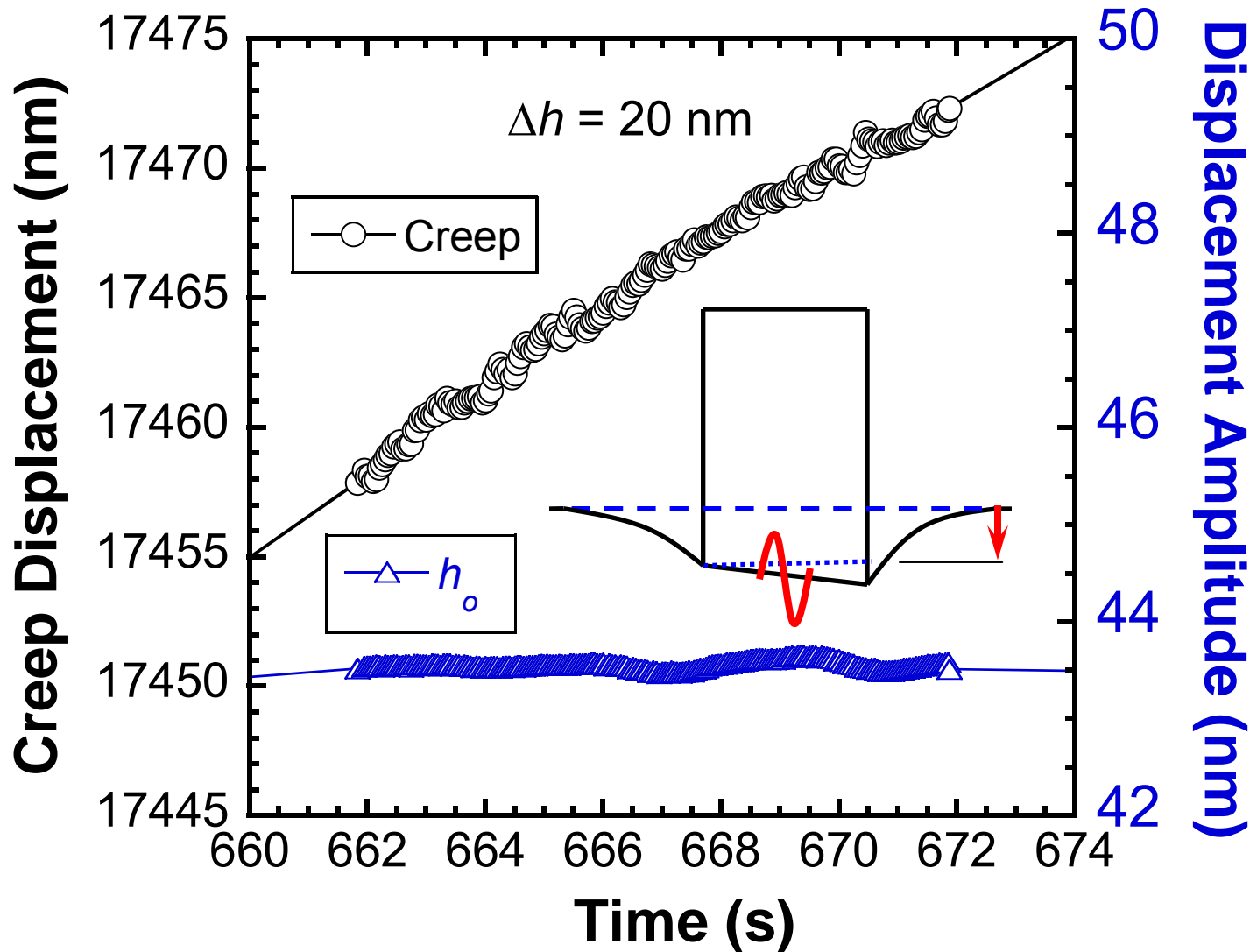


# CLOSING REMARKS

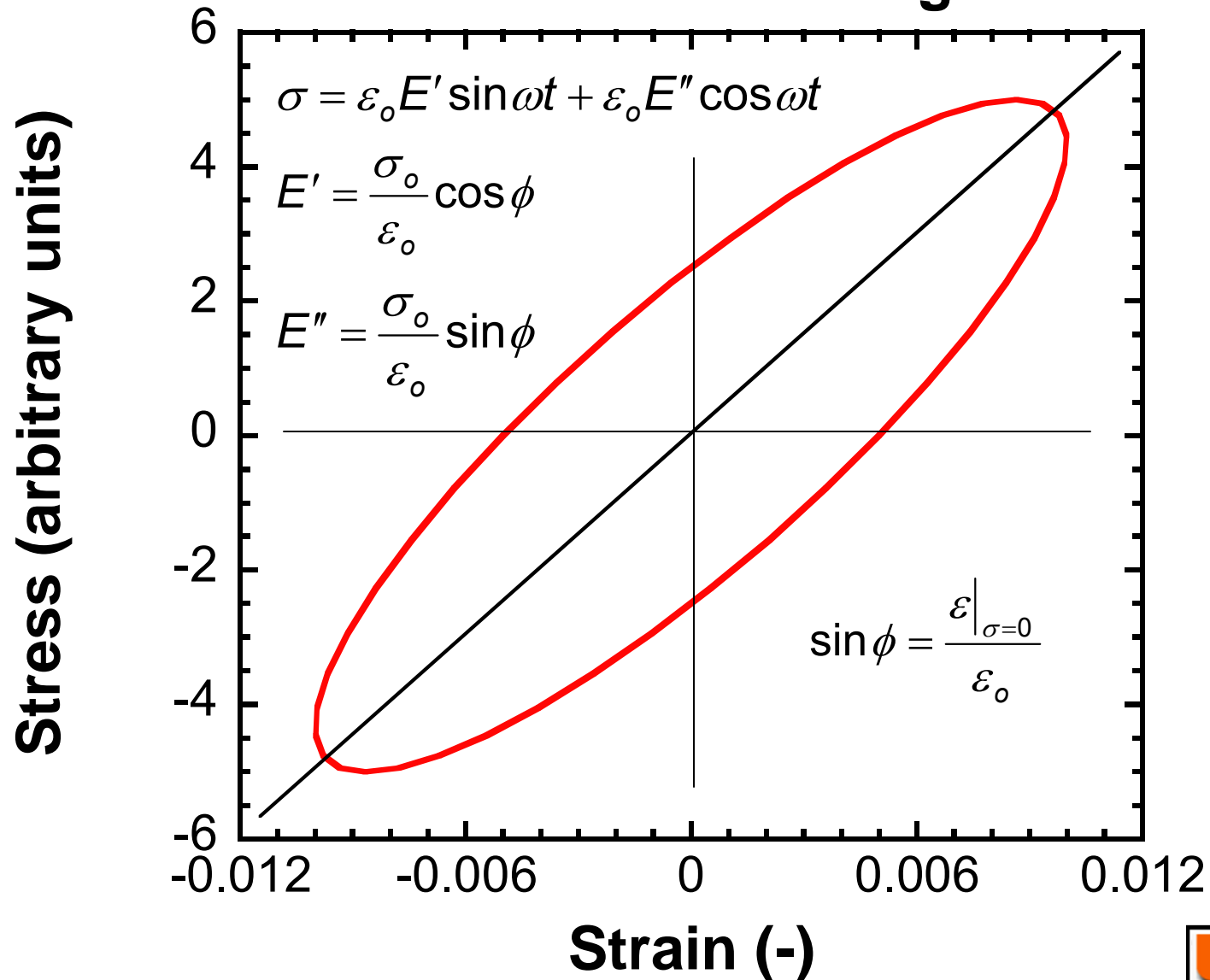
- ❖ Goals of a dynamic nanoindentation experiment:
  - Robust dynamic characterization of the instrument
  - Known contact geometry
  - Steady-state harmonic motion
  - Linear viscoelasticity
- ❖ If these criterion are met, dynamic nanoindentation can be used to make accurate and precise measurements of the complex modulus.
- ❖ These experiments can be performed in a completely automated fashion, in 90 seconds or less, and with a 100% success rate – no failed measurements.



# Steady-State Harmonic Motion



# Linear viscoelasticity, sinusoidal loading



# Linear viscoelasticity, sinusoidal loading

